SOLUCION EXAMEN 1 - 2º CH.

Matrices y determinantes

1 a) $\left(\begin{array}{ccc|c} 1 & 2 & 1 & a \\ 1 & 1 & -a & a \\ 2 & 3 & 1 & a \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & a \\ 0 & 1 & 1+a & 0 \\ 0 & 1 & 1 & a \end{array}\right) \sim$

$\sim \left(\begin{array}{ccc|c} 1 & 2 & 1 & a \\ 0 & 1 & 1+a & 0 \\ 0 & 0 & a & -a \end{array}\right)$ Fin Gauss

Raíces $\begin{cases} a=0 \\ 1+a=0 \Rightarrow a=-1 \end{cases}$

Si $a=0$ $\left(\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right)$ Rango A = Rango $A^* = 2$ por R.F. compatible indeterminado

Si $a=-1$ $\left(\begin{array}{ccc|c} 1 & 2 & 1 & -1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & -1 \end{array}\right)$ Rango A = Rango $A^* = 3$ por R.F. compatible determinado

$\therefore \forall a \neq 0$ compatible determinado

SOLUCION:

$za = -a \Rightarrow \boxed{z = -1}$

$y + (1+a)z = 0 \quad \boxed{y = 1+a}$

$x + 2y + z = a \Rightarrow x + 2(1+a) - 1 = a$

$x = a + 1 - 2 - 2a = \boxed{-(1+a)}$

SOLUCION $a=0$ S.C. Indeterminado

$\boxed{z = \lambda}$

$y + z = 0 \Rightarrow \boxed{y = -\lambda}$

$x + 2y + z = 0 \Rightarrow x - \lambda = 0 \quad \boxed{x = \lambda}$

$\forall \lambda \in \mathbb{R}$

▨

2)

a) $M = \begin{pmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 2 \end{pmatrix} \quad \begin{pmatrix} + & - & + \\ - & + & - \\ + & - & + \end{pmatrix}$

$|M| = 2 + 4 + 1 - 2 = 5 \neq 0$ Regular $\therefore$ Inversible

$(Adj\,M) = \begin{pmatrix} 0 & -5 & 5 \\ 2 & 3 & -2 \\ -1 & 1 & 1 \end{pmatrix} \Rightarrow A^{-1} = \dfrac{(adj\,M)^t}{|A|} = \begin{pmatrix} 0 & 2/5 & -1/5 \\ -1 & \frac{3}{5} & 1/5 \\ 1 & -2/5 & 1/5 \end{pmatrix}$

b) $\left.\begin{array}{l} 8A - 5B = C \\ 2A - B = D \end{array}\right)$ $\quad \begin{array}{r} 8A - 5B = C \\ -8A + 4B = -4D \\ \hline -B = C - 4D \end{array}$

$\boxed{B = 4D - C}$

$2A + C - 4D = D$

$2A = 5D - C$

$\boxed{A = \frac{1}{2}(5D - C)}$

$4D = \begin{pmatrix} 4 & -16 & 0 \\ 8 & -4 & 12 \\ 0 & 4 & -4 \end{pmatrix} \quad 5D = \begin{pmatrix} 5 & -20 & 0 \\ 10 & -5 & 15 \\ 0 & 5 & -5 \end{pmatrix}$

$-C = \begin{pmatrix} -3 & -2 & 0 \\ 2 & -1 & -3 \\ 0 & -3 & 3 \end{pmatrix}$

$A = \frac{1}{2}\begin{pmatrix} 2 & -22 & 0 \\ 12 & -6 & 12 \\ 0 & 2 & -2 \end{pmatrix} = \begin{pmatrix} 1 & -11 & 0 \\ 6 & -3 & 6 \\ 0 & 1 & -1 \end{pmatrix}$ ▨

$B = \begin{pmatrix} 1 & -18 & 0 \\ 10 & -5 & 9 \\ 0 & 1 & -1 \end{pmatrix}$ ▨

[redacted]

a) $A = \begin{pmatrix} a & 0 & 2a \\ a & a-1 & 0 \\ -a & 0 & -a \end{pmatrix}$

Sarrus

$|A| = -a^2(a-1) + 2a^2(a-1) = a^2(a-1)$

Raíces $\begin{cases} 0 \\ 1 \end{cases}$ $\therefore$ $\forall a \neq 0$ y 1 Determinante no nulo

$|2A| = 2^3\,|A| = 8a^2(a-1)$

Si $a=3 \Rightarrow |2A| = 8.9.2 = 144$

b): propiedades $\begin{vmatrix} 3a & 3b & 3c \\ a+p & b+q & c+r \\ -x+a & -y+b & -z+c \end{vmatrix} =$

$3\begin{vmatrix} a & b & c \\ a+p & b+q & c+r \\ -x+a & -y+b & -z+c \end{vmatrix} = 3\left(\underbrace{\begin{vmatrix} a & b & c \\ a & b & c \\ -x+a & -y+b & -z+c \end{vmatrix}}_{=0\ (2\text{ filas iguales})} + \begin{vmatrix} a & b & c \\ p & q & r \\ -x+a & -y+b & -z+c \end{vmatrix}\right) =$

$= 3\left(\begin{vmatrix} a & b & c \\ p & q & r \\ -x & -y & -z \end{vmatrix} + \underbrace{\begin{vmatrix} a & b & c \\ p & q & r \\ a & b & c \end{vmatrix}}_{=0\ (2\text{ filas iguales})}\right) = -3\begin{vmatrix} a & b & c \\ p & q & r \\ x & y & z \end{vmatrix} = -3 \cdot 7 = \underline{\underline{-21}}$